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Proton-Nucleus Total Cross Sections in Coupled-Channel Approach

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Abstract

Recently, nucleon-nucleon (N-N) cross sections in the medium have been extracted directly from experiment. The in-medium N-N cross sections form the basic ingredients of several heavy-ion scattering approaches including the coupled-channel approach developed at the Langley Research Center. In the present study the ratio of the real to the imaginary part of the two-body scattering amplitude in the medium was investigated. These ratios are used in combination with the in-medium N-N cross sections to calculate total proton-nucleus cross sections. The agreement is excellent with the available experimental data. These cross sections are needed for the radiation risk assessment of space missions.

Introduction

The transportation of energetic ions in bulk matter is of direct interest in several areas including shielding against ions originating from either space radiations or terrestrial accelerators, cosmic ray propagation studies in galactic medium, or radiobiological effects resulting from the work place or clinical exposures. For carcinogenesis, terrestrial radiation therapy, and radiobiological research, knowledge of beam composition and interactions is necessary to properly evaluate the effects on human and animal tissues. For the proper assessment of radiation exposures both reliable transport codes and accurate input parameters are needed. One such important input is total cross sections. The motivation of the work is to develop a method for calculating accurate cross sections. These cross sections are needed in transport methods both deterministic and Monte Carlo.

Nucleon-nucleon (N-N) cross sections are the basic ingredients of many approaches (refs. 1 to 10) to heavy ion scattering problem. Most of the information about these N-N cross sections comes from the free two-body scattering. These cross sections are significantly modified in a nucleus, due to the presence of other nucleons, which is affected through the Pauli exclusion principle and modification of meson field coupling constants. (See ref. 11.) Our theoretical approach is based on the coupled channel method used at the Langley Research Center. (See refs. 1 to 6.) This method solves the Schrödinger equation in eikonal approximation. The method needs modifications at low and medium energies. In earlier work (refs. 12 and 13), the renormalization of the free N-N cross sections were found to be significant at lower and medium energies. These modified in-medium N-N cross sections were used to calculate the total reaction cross sections. The blend of the renormalized N-N cross sections and the coupled-channel method were found to give a reliable basic approach to total reaction cross sections. The purposes of the current paper are as follows:

- (1) To investigate the modification of the ratio of the real to the imaginary part of the two-body amplitude in the medium
- (2) To use these modified two-body amplitudes to calculate total cross sections for proton-nucleus collisions
- (3) To validate and compare the calculated results with the available experimental data
- (4) To provide theoretical results where data are not available because of nonexistence of experimental facilities and/or difficulty in experimental data analysis

Method

The essentials of the coupled-channel method are briefly sketched for completeness. (See refs. 1 to 6 for details.) In this approach, the matrix for elastic scattering amplitude is given by

$$f(\mathbf{q}) = -\frac{ik}{2\pi} \int d^2\mathbf{b} \exp(-i\mathbf{q} \cdot \mathbf{b}) \{ \exp[i\chi(\mathbf{b})] - 1 \} d^2\mathbf{b} \quad (1)$$

where

f and χ matrices

k projectile momentum relative to center of mass

\mathbf{b} projectile impact parameter vector

\mathbf{q} momentum transfer

$\chi(\mathbf{b})$ eikonal phase matrix

The total cross section σ_{tot} is found from the elastic scattering amplitude by using the optical theorem as follows:

$$\sigma_{\text{tot}} = \frac{4\pi}{k} \text{Im} [f(q=0)] \quad (2)$$

Equations (1) and (2) give

$$\sigma_{\text{tot}} = 4\pi \int_0^\infty b db \{ 1 - e^{-\text{Im}(\chi)} \cos [\text{Re}(\chi)] \} \quad (3)$$

The eikonal phase matrix χ (see refs. 1 to 6 for details) is given by

$$\chi(\mathbf{b}) = \chi_{\text{dir}}(\mathbf{b}) - \chi_{\text{ex}}(\mathbf{b}) \quad (4)$$

The direct and exchange terms are calculated with the following expressions (refs. 1 to 6):

$$\chi_{\text{dir}}(\mathbf{b}) = \frac{A_P A_T}{2\pi k_{NN}} \int d^2\mathbf{q} \exp(i\mathbf{q} \cdot \mathbf{b}) F^{(1)}(-\mathbf{q}) G^{(1)}(\mathbf{q}) f_{NN}(\mathbf{q}) \quad (5)$$

$$\begin{aligned} \chi_{\text{ex}}(\mathbf{b}) &= \frac{A_P A_T}{2\pi k_{NN}} \int d^2\mathbf{q} \exp(i\mathbf{q} \cdot \mathbf{b}) F^{(1)}(-\mathbf{q}) G^{(1)}(\mathbf{q}) \\ &\times \frac{1}{(2\pi)^2} \int d^2\mathbf{q}' \exp(i\mathbf{q}' \cdot \mathbf{b}) f_{NN}(\mathbf{q} + \mathbf{q}') C(\mathbf{q}') \end{aligned} \quad (6)$$

where

$F^{(1)}$ and $G^{(1)}$ projectile and target ground-state one-body form factors, respectively

k_{NN} relative wave number in two-body center-of-mass system

C correlation function (ref. 6)

A_P and A_T mass numbers of projectile and target nuclei, respectively

The two-body amplitude f_{NN} is parameterized as

$$f_{NN} = \frac{\sigma(\alpha + i)}{4\pi} k_{NN} \exp\left(-\frac{B\mathbf{q}^2}{2}\right) \quad (7)$$

where

σ two-body cross section

B slope parameter

α ratio of real part to imaginary part of forward, two-body amplitude

It is well-known that the absorption cross section depends on the imaginary part of the eikonal phase matrix. This leads us to write the two-body amplitude in the medium $f_{NN,m}$ as

$$f_{NN,m} = f_m f_{NN} \quad (8)$$

where f_{NN} is the free NN amplitude and f_m is the system and energy dependent medium multiplier function. (See refs. 12 and 13.) Then the nucleon-nucleon cross sections in the medium ($\sigma_{NN,m}$) can be written as

$$\sigma_{NN,m} = f_m \sigma_{NN} \quad (9)$$

where σ_{NN} is the nucleon-nucleon cross section in free space and the medium multiplier is given by

$$f_m = 0.1 \exp(-E/12) + \left[1 - \left(\frac{\rho_{av}}{0.14} \right)^{1/3} \exp\left(-\frac{E}{D}\right) \right] \quad (10)$$

where E is the laboratory energy in units of A MeV, D is a parameter in units of MeV, as defined subsequently. The numbers 12 and 0.14 are in units of MeV and fm^{-3} , respectively. For $A_T \leq 56$ (mass number for iron ion representing heavy elements considered in our transport phenomena),

$$D = 46.72 + 2.21 A_T - (2.25 \times 10^{-2}) A_T^2 \quad (11)$$

and for $A_T > 57$,

$$D = 100 \text{ MeV} \quad (12)$$

In equation (10), ρ_{av} refers to the average density of the colliding system and is

$$\rho_{av} = \frac{1}{2} (\rho_{AP} + \rho_{AT}) \quad (13)$$

where the density of a nucleus A_i ($i = P, T$) is calculated in the hard sphere model and is given by

$$\rho_{A_i} = \frac{A_i}{(4\pi/3) r_i^3} \quad (14)$$

where the radius of the nucleus r_i is defined by

$$r_i = 1.29 (r_i)_{\text{rms}} \quad (15)$$

The root-mean-square radius $(r_i)_{\text{rms}}$ is obtained directly from experiment (ref. 14) after subtraction of the nucleon charge form factor (ref. 2).

From equation (3), note that total cross section depends on real component of eikonal phase matrix and, hence (eqs. (5), (6), and (7)), on the product of $\sigma\alpha$ in two-body amplitude. Since the modification of the cross sections in the medium have been determined and tested thoroughly (refs. 12 and 13), the modification of α , ratio of the real to the imaginary part of the two-body amplitude, is studied in the medium to calculate the total cross sections. Some data for total cross sections are available for a few systems at high energies. Unfortunately, no data are available for total cross sections in the low and medium energy range; there are some data for $p + Pb$ in the 100 A MeV range. Therefore, values of the medium-modified α have been tested for higher energies. At low and medium energies, our theoretical results, which incorporate the in-medium two-body amplitudes, can be validated, if and when experimental data become available.

A best estimate of medium-modified α takes into account the enhancement of the cross sections (ref. 15) and stability and is given by

$$\alpha_m = 3 \exp \left[\frac{-\left(E - 13A^{1/3}\right)^2}{5000} \right] + \frac{K}{1 + \exp[(10 - E)/75]} \quad (16)$$

where

$$K = 0.35 + 0.65 \exp \left[-\frac{2}{3}(N - Z) \right] \quad (17)$$

with N being the neutron number of the nucleus and Z its charge number.

Equation (3) has also been modified to account for the Coulomb force in the proton-nucleus cross sections. This modification has significant effects at low energies and becomes less important as the

energy increases and practically disappears for energies around 50 A MeV and higher.

For nucleus-nucleus collisions, the Coulomb energy is given by

$$V_B = \frac{1.44 Z_P Z_T}{R} \quad (18)$$

where the constant 1.44 is in units of MeV-fm, Z_P and Z_T are charge numbers for the projectile and target, respectively, and R , the radial distance between their centers, is given by

$$R = r_P + r_T + 1.2 \frac{A_P^{1/3} + A_T^{1/3}}{E_{CM}^{1/3}} \quad (19)$$

The number 1.2 in equation (19) is in units of fm-MeV^{1/3}. In our earlier work (refs. 12 and 13), these expressions were used also for the proton-nucleus collisions in order to have a unified picture of any colliding system. However, as shown in the references, equation (19) overestimates the radial distance between proton-nucleus collisions, and hence, equation (18) underestimates the Coulomb energy between them. To compensate for this, we multiplied equation (18) by the following factor (refs. 12 and 13), which gives the Coulomb multiplier to equation (3)

$$X_{Coul} = \left(1 + \frac{C_1}{E_{cm}}\right) \left(1 - \frac{C_2 V_B}{E_{cm}}\right) \quad (20)$$

For $A_T \leq 56$ (mass number for iron),

$$\begin{aligned} C_1 &= 6.81 - 0.17 A_T + (1.88 \times 10^{-3}) A_T^2 \\ C_2 &= 6.57 - 0.30 A_T + (3.6 \times 10^{-3}) A_T^2 \end{aligned} \quad (21)$$

The constant C_1 is in units of MeV. For $A_T > 57$,

$$\begin{aligned} C_1 &= 3.0 \text{ MeV} \\ C_2 &= 0.8 \end{aligned} \quad (22)$$

For the nucleus-nucleus collisions,

$$C_1 = 0 \text{ MeV}$$

$$C_2 = 1$$

This form of Coulomb energy was found to work well for the proton-nucleus absorption cross sections (ref. 12). Equation (3) is the main equation and is multiplied by equation (20) to get the total cross sections shown in figures 1 to 6.

Results and Conclusions

Figures 1 to 6 show the results of our calculations for the total cross sections for proton on beryllium, carbon, aluminum, iron, lead, and uranium targets, respectively. The experimental data have been taken from the compilation of references 16 and 17. There is paucity of data at lower and intermediate energies (there are some data for $p + Pb$ in the 100 A MeV range), where the medium modifications play a significant role. For the energy ranges considered, where the data are unavailable, our results provide good theoretical values of total cross sections, since many renormalization effects due to medium, which play an important role in cross sections, have been incorporated in the formalism.

Very good agreement with the experimental results is found for all the systems at higher energies where some data are available. We note that the in-medium cross sections derived earlier in combination with the modified ratio of the real to the imaginary part of the amplitude provide good results for the proton-nucleus total cross sections. It is gratifying to note that the present method gives a consistent basic approach for the total reaction and the total cross sections for the entire energy range for all the systems studied here.

The in-medium two-body amplitudes developed in our approach can be used with ease in other nuclear processes as well.

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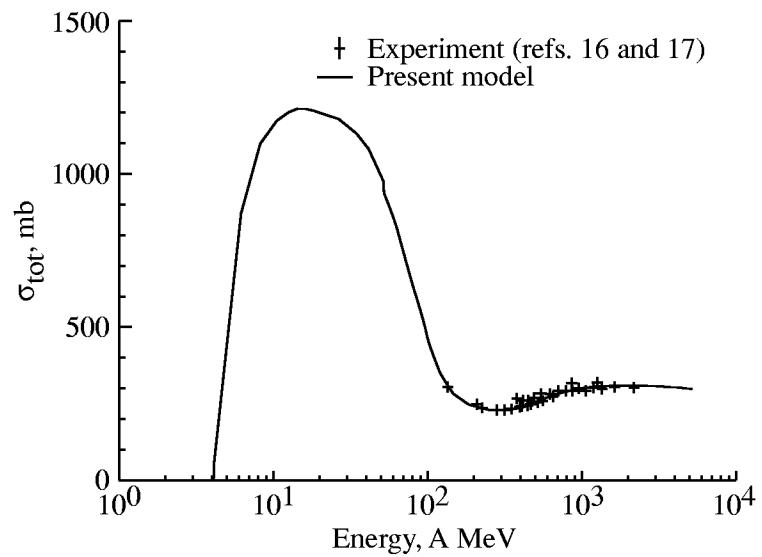


Figure 1. Total cross sections for proton-beryllium collision as function of energy.

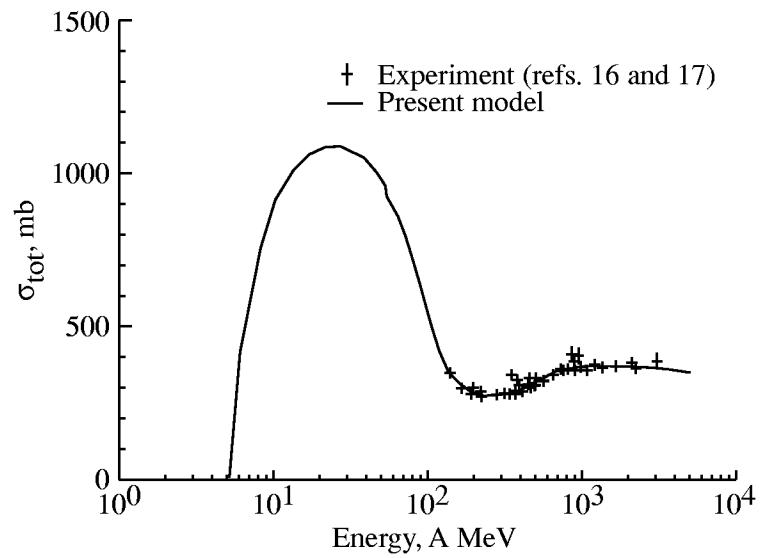


Figure 2. Total cross sections for proton-carbon collision as function of energy.

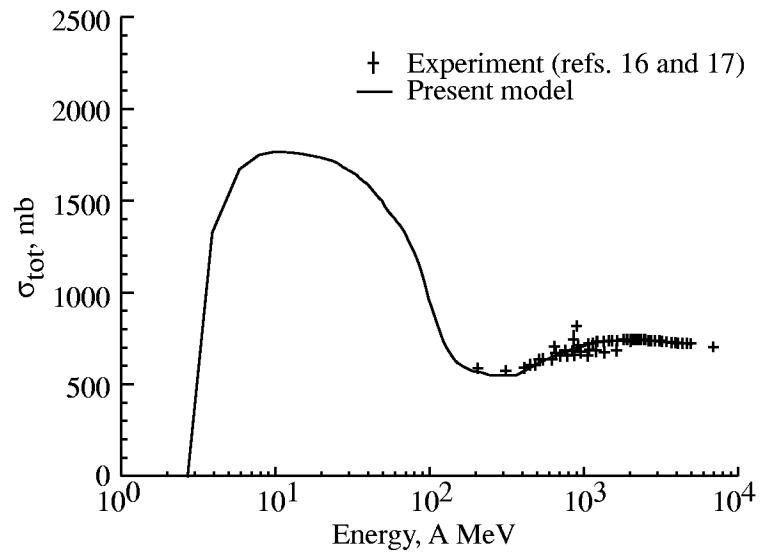


Figure 3. Total cross sections for proton-aluminum collision as function of energy.

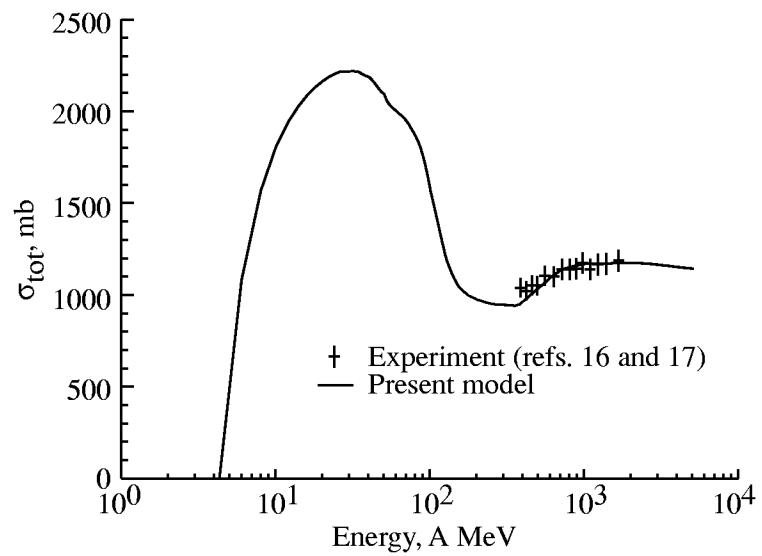


Figure 4. Total cross sections for proton-iron collision as function of energy.

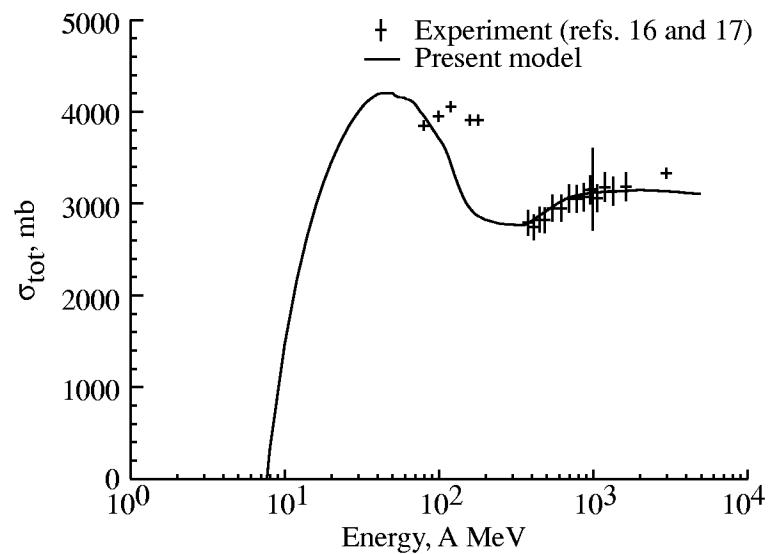


Figure 5. Total cross sections for proton-lead collision as function of energy.

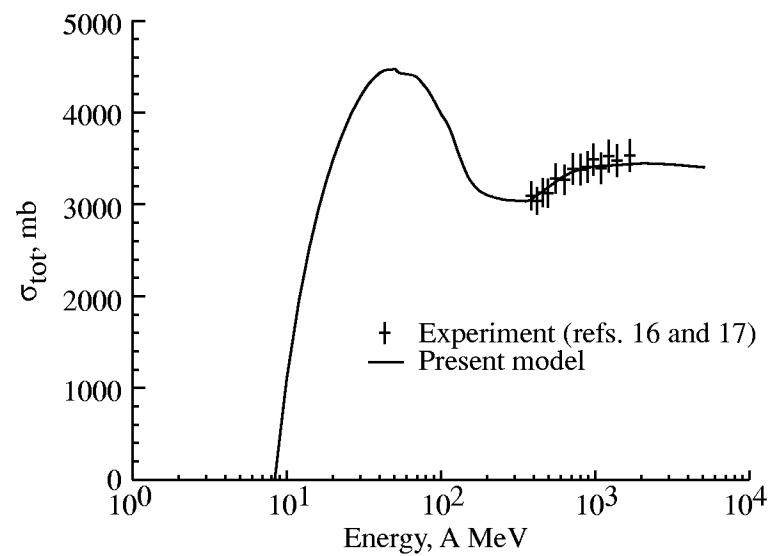


Figure 6. Total cross sections for proton-uranium collision as function of energy.

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